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# Regulation of Settling Chamber Pressure in a Hypersonic Wind Tunnel using LQR Controller

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***Abstract* — Wind tunnels are used to study the aerodynamic properties of space specimen under test. Hypersonic wind tunnel is used to simulate flight conditions of space vehicles in hypersonic flow regime. This paper aims to design and analyze the performance of a Linear Quadratic Regulator (LQR) based controller for the regulation of pressure inside the settling chamber of a hypersonic wind tunnel. The design is carried out using optimal state feedback controller gain matrix of the linear model of the tunnel system which is completely state controllable and observable. Simulation results show that the change in set point does not affect the settling time whereas the peak overshoot increases with increase in set point. The settling time as well as the peak overshoot are improved when the value of weighing matrix is increased.**

***Keywords* – Hypersonic Wind Tunnel, Linear-Quadratic Regulator, Settling Chamber Pressure.**

## I. Introduction

Hypersonic wind tunnel is a ground based test facility to study the aerodynamic properties of space vehicles, race cars, fighter planes placed in the hypersonic flow regime. These tunnels operate at hypersonic speeds with mach number greater than 5, which is defined as ratio of speed of aircraft to speed of sound in gas [1], [2]. The different subsystems of the hypersonic wind tunnel are high pressure system(HP), vacuum system(Vac), Pressure regulating valve(PRV), Heater(H1), Settling chamber(SC), Nozzle(NOZ) and Test section(TS) as shown in fig.(1).

Air which is compressed and stored in high pressure system is released through a pressure valve to heater where it is heated to the required temperature and is passed to the settling chamber. The pressure in settling chamber must be suitably controlled so that flow through test section meets the mach number and mass flow rate required for testing and thereby meets the performance requirement [3]-[6].

LQR controller as the name suggests requires a linear model of the system for which it will generate constant gains for full state feedback to make the equilibrium point globally asymptotically stable. As the word regulator suggests, the controller is used to track the output and follow the changes in the set point. LQR controller is an optimization-based synthesis problem taking into account performance, control energy and performance requirements of the system under consideration [7], [8].

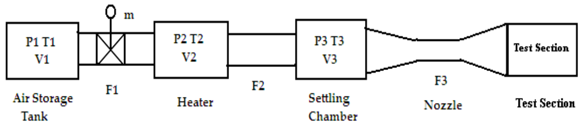


Fig.1. Block diagram of hypersonic wind tunnel.

## II. Model of the System

The tunnel system is divided into three pressure vessels and the continuity equation of pressure vessel and parameter values are used to develop the model [9]-[11], assuming the temperature remains constant for run time. The state space model of the system is given in (1) and (2).

$$\begin{bmatrix}\dot{P}_1\\ \dot{P}_2\\ \dot{P}_3\end{bmatrix} = \begin{bmatrix}-K_1/C_1 & 0 & 0\\ K_1/C_1 & -K_3/C_2 & -K_4/C_2\\ 0 & K_3/C_3 & K_4-K_n/C_3\end{bmatrix}\begin{bmatrix}P_1\\ P_2\\ P_3\end{bmatrix} + \begin{bmatrix}-K_2/C_1\\ K_2/C_2\\ 0\end{bmatrix} m \qquad (1)$$

$$Y_1 = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}P_1\\ P_2\\ P_3\end{bmatrix} \qquad (2)$$

Where $P_1$, $P_2$ are the upstream and downstream pressures, $P_3$ is the settling chamber pressure, m is the stem movement of pressure valve, $K_1$, $K_2$, $K_3$, and $K_4$ are constants which depends on the flow rate, pressure in the three vessels, and stem movement [9]. Kn is the nozzle flow constant and, $C_1$, $C_2$, $C_3$ represents the capacitance of the three pressure vessels respectively. The generalised transfer function of the system [9] is given by

$$G_p(s) = \frac{-2.369e006s^2+7.897e007s+4.21e005}{0.015s^5+0.7802s^4+9.89s^3+18.46s^2+3.377s+0.01937} \qquad (3)$$

The temperature in the pressure vessels 1 $T_1$, heater $T_2$ and vessel 3, $T_3$ are varied according to the operating conditions obtained from empirical analysis [9]. Six operating conditions are selected for perturbation studies.

Case1. $P_1 = 300\times10^5$ Pa, $T_1 = 300^0$K, $P_2 = 77\times10^5$ Pa, $T_2 = 700^0$K, $P_3 = 70\times10^5$ Pa, $T_3 = 539^0$K
Case2. $P_1 = 235\times10^5$ Pa, $T_1 = 271^0$K, $P_2 = 77\times10^5$ Pa, $T_2 = 650^0$K, $P_3 = 70\times10^5$ Pa, $T_3 = 529^0$K
Case3. $P_1 = 300\times10^5$ Pa, $T_1 = 300^0$K, $P_2 = 44\times10^5$ Pa, $T_2 = 700^0$K, $P_3 = 40\times10^5$ Pa, $T_3 = 518^0$K
Case4. $P_1 = 250\times10^5$ Pa, $T_1 = 278^0$K, $P_2 = 44\times10^5$ Pa, $T_2 = 650^0$K, $P_3 = 40\times10^5$ Pa, $T_3 = 508^0$K
Case5. $P_1 = 300\times10^5$ Pa, $T_1 = 300^0$K, $P_2 = 100\times10^5$ Pa, $T_2 = 700^0$K, $P_3 = 100\times10^5$ Pa, $T_3 = 554^0$K
Case6. $P_1 = 252\times10^5$ Pa, $T_1 = 278^0$K, $P_2 = 100\times10^5$ Pa, $T_2 = 650^0$K, $P_3 = 100\times10^5$ Pa, $T_3 = 544^0$K

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## III. Analysis of System Model

### A. Stability Analysis

By substituting the values of the parameters $K_1$, $K_2$, $K_3$, $K_4$, Kn, $C_1$, $C_2$, and $C_3$ from [9] in (1) and (2), the state model is obtained with

$$[A] = \begin{bmatrix} -0.0045 & 0 & 0 \\ 0.0045 & 2.51 & 2.51 \\ 0 & 12.85 & -14.14 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 617679.68 \\ 6133259.91 \\ 0 \end{bmatrix}$$

$$[C] = [0\ 0\ 1]$$

Controllability and observability tests are carried out on the model using Kalman's test [12] before designing the controller. The tests are carried out using (4) and (5) respectively.

$$Q_C = [B\ AB\ A^2B] \quad (4)$$

$$Q_O = \left[C^T\ A^TC^T\ A^{T^2}C^T\right] \quad (5)$$

It is found that $|Q_C| = -1.26 * 10^{22} \neq 0$ and $|Q_O| = -743.05 * 10^{-3} \neq 0$ and rank of the matrix is 3, which is equal to the dimension of the system and hence the system is completely state controllable and observable [12].

### B. Open Loop Response of System

The linear model of the hypersonic wind tunnel system using (1)-(3) is simulated in Matlab. The physical and model parameters for simulation are selected for the system [9]-[11]. The open loop response of the settling chamber pressure $P_3$ is shown in fig.(2).

It is observed from the figure that the peak value of settling chamber pressure is $130\times10^5$ Pa. From the figure, it can also be observed that the corresponding settling time is 450sec which is quite high when the test duration is very short.

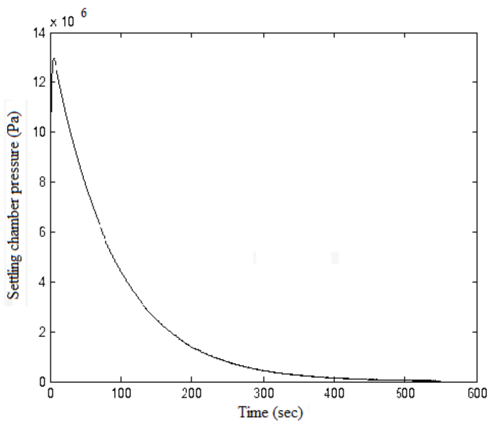


Fig.2. Open Loop Response of the System.

## IV. Design of LQR Controller

LQR is an optimal control scheme which provides a systematic way of calculating the state feedback control gain matrix, K. LQR controller design problem deals with optimizing an energy function, J by designing the state feedback controller, K.

A system in state variable form is

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

with x(t) ε $R^n$ and u(t) ε $R^m$ . x is the state of the system and u is the control input. The initial condition is x(0) and states are measurable. The state-variable feedback (SVFB) control law is

u = -K x

where K is the linear optimal feedback control gain matrix [13]. The closed-loop system using this control becomes

$$\dot{x} = (A - BK)x$$

Where v is the new command input. In LQR controller design, the crucial property is that this closed-loop is asymptotically stable as long as the system is controllable and observable. It is noted that the output matrices C and D are not used in SVFB design. The objective is to find the optimal control law that minimizes the following performance index. The performance index (PI) [8], [13], [14] is defined by

$$J = \frac{1}{2}\int_0^\infty (x^TQX + u^TRu)\,dt \quad (6)$$

The performance index, $J$ is the energy function which keeps the total energy of the closed-loop system small. The two matrices Q and R are selected such that Q is positive semi-definite and R is positive definite [7], [13]. The control value u is called optimal control [13] which is given by,

$$u(t) = -R^{-1}B^TPx = -Kx \quad (7)$$

where P(t) is the solution of Riccati equation and is a real symmetric matrix. Solving (7)

$$PA + A^TP - PBR^{-1}BP + Q = 0 \quad (8)$$

K is obtained as

$$K = R^{-1}B^TP$$

The plant with LQR controller is shown in fig.(3).

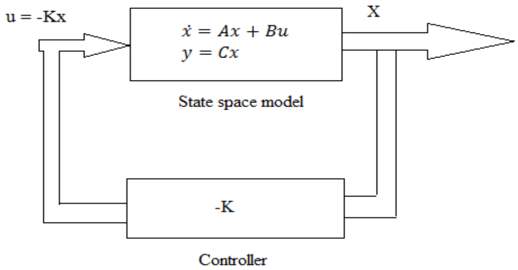


Fig.3. Full state feedback representation of the system.

The effect of optimal control using LQR controller depends on proper selection of weight matrices Q and R. The approach used for selecting Q and R is trial method by simulation which helps to find optimal gain matrix, K.

## V. Results And Analysis of LQR Controller

The gain matrix, K satisfying the control law for the LQR is estimated using Matlab. After selecting the weight matrix Q and R as

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$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R = 1,

the optimal gain matrix K is obtained.

K = [-0.9839 1.1041 0.3943].

With these values of gain, the system is simulated to get the response of settling chamber pressure with three different values of set points. Fig.(4) show the variation of settling chamber pressure with LQR controller for set points of $50 * 10^5$ Pa, $80 * 10^5$ Pa, $100 * 10^5$ Pa respectively.

The performance parameters and various error index are also evaluated and the results are tabulated in table(1). From the response, it is observed that the change in set point does not affect the settling time and rise time whereas the peak overshoot increases drastically with increase in set point. Here the settling time is 18 secs and the rise time is 2 secs. The percentage peak overshoot values are 32, 44, and 76.31corresponding to the three set points. The error index Integral of absolute error (IAE), integral of square of errors (ISE), integral of time absolute errors (ITAE) show similar trends with increase in set point values.

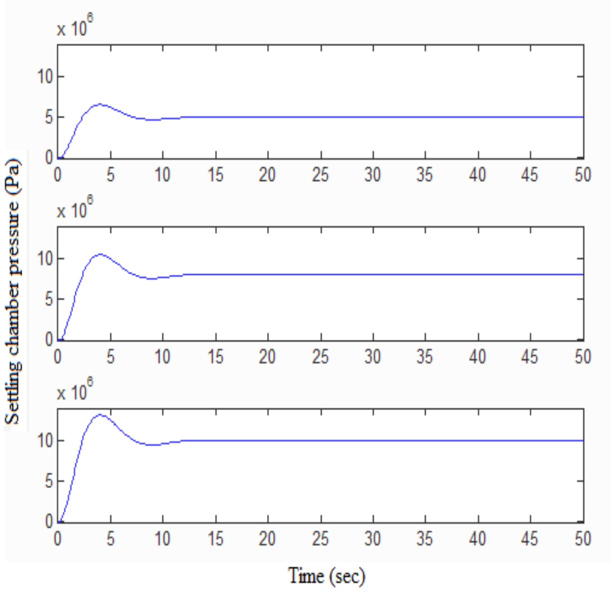


Fig.4. Settling chamber pressure with LQR controller for set point $50 * 10^5, 80 * 10^5, 100 * 10^5$ Pa and Q = (1 0 0 0 1 0 0 0 1)

The effect of change of weighing matrix, Q on the performance is evaluated by selecting Q as

$$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

Corresponding optimal gain matrix K

K = [ –10.2745 11.0853 3.9464].

The system is simulated using these values and the response of settling chamber pressure for same set points are plotted. Fig.(5) shows the variation of settling chamber pressure with LQR controller for set points of $50 * 10^5$ Pa, $80 * 10^5$ Pa, $100 * 10^5$ Pa respectively.

Table I: Parameters for settling chamber pressure using LQR controller with Q = (1 0 0 0 1 0 0 0 1)

| Performance Of Hypersonic Wind Tunnel With Lqr Controller When Q = (1 0 0 0 1 0 0 0 1) | | | |
|---|---|---|---|
| Parameters | Set point = $50 * 10^5$ Pa | Set point = $80 * 10^5$ Pa | Set point = $100 * 10^5$ Pa |
| Settling Time | 18 secs | 19 secs | 18 secs |
| Rise Time | 2secs | 2secs | 2secs |
| Peak Overshoot | 32% | 44% | 76.31% |
| IAE | $1.0*10^7$ | $1.6*10^7$ | $2.0*10^7$ |
| ISE | $5.0*10^{13}$ | $1.28*10^{14}$ | $2.0*10^{14}$ |
| ITAE | $5.87*10^3$ | $1.47*10^4$ | $2.29*10^{14}$ |

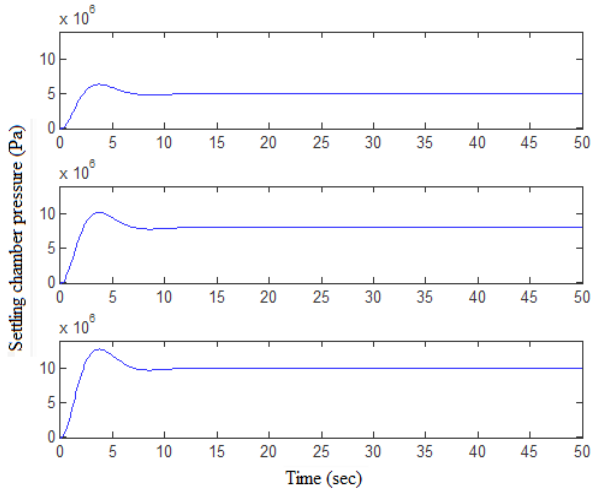


Fig.5. Settling chamber pressure with LQR controller for set point $50 * 10^5, 80 * 10^5, 100 * 10^5$ Pa and Q = (100 0 0 0 100 0 0 0 100)

The various performance parameters and the corresponding error index are also calculated and the results are tabulated in table(2). It is evident from the table that the settling time and rise time does not change with variation of set point. However the peak overshoot show a slight increases with increase in set points. The error index corresponding to three set points are also evaluated.

The effect of variation of weighing matrix, Q is clear from these analysis. When the value of Q is changed from (1 0 0 0 1 0 0 0 1) to (100 0 0 0 100 0 0 0 100), the settling time is reduced from 18 secs to 15 secs whereas the rise time remains the same. The main advantage is the drastic reduction in percentage peak overshoot. The error index IAE and ISE values remains the same for both the values of weighing matrix, Q whereas the ITAE shows slight deviations. The settling time in the case of open loop response is 450 secs which is improved to 18 secs with LQR controller with Q value (1 0 0 0 1 0 0 0 1) and 15 secs with Q value (100 0 0 0 100 0 0 0 100).

Table II: Parameters for settling chamber pressure using LQR controller with Q = (100 0 0 0 100 0 0 0 100)

| PERFORMANCE OF HYPERSONIC WIND TUNNEL WITH LQR CONTROLLER WHEN Q = (100 0 0 0 100 0 0 0 100) | | | |
|---|---|---|---|
| Parameters | Set point = 50 * $10^5$ Pa | Set point = 80 * $10^5$ Pa | Set point = 100 * $10^5$ Pa |
| Settling Time | 15 secs | 15 secs | 15 secs |
| Rise Time | 2secs | 2secs | 2.5secs |
| Peak Overshoot | 25.38% | 26.25% | 29% |
| IAE | $1.0*10^7$ | $1.6*10^7$ | $2.0*10^7$ |
| ISE | $5.0*10^{13}$ | $1.28*10^{14}$ | $2.0*10^{14}$ |
| ITAE | $6.09*10^{13}$ | $1.50*10^{14}$ | $2.34*10^{14}$ |

## VI. Conclusion

An LQR based controller is designed for regulating the settling chamber pressure of a hypersonic wind tunnel system. The model of the tunnel system is subjected to stability analysis using Kalman's test before designing the controller. The open loop response of the system is plotted. The performance of the system is evaluated for increasing values of three set points. The effect of variation of weighing matrix on the system performance is also analyzed.

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